The conditional dependence structure among precious metals: a copula-GARCH approach

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Abstract. The aim of the paper is to analyse the conditional dependence structure among precious metal returns using a copula-DCC-GARCH approach. Conditional correlation matrices are used to identify the states of the precious metals market by assuming that a given state of the market corresponds to a typical pattern of the conditional dependence structure. Cluster analysis allows for pointing at transition points among the market states, that is the points of drastic change in the conditional dependence structure. The application of the methodology described above to the period between 1997 and 2013 indicates three market states of four major precious metals (gold, silver, platinum and palladium). The results obtained reveal a sudden increase in dependencies among precious metals at the turn of April and May 2004.

Keywords: precious metals, dependence structure, copula-GARCH, market states.

JEL Classification: C58, Q02
AMS Classification: 91G70, 62P20

1 Introduction

The existing research on precious metals focuses mainly on gold and silver. The analysis of the prices of precious metals can be divided into two areas. The first one covers the analysis of the relationship among prices of precious metals. Ciner [2] finds evidence of the disappearance of the long term relationship between gold and silver in the 1990s. Their conclusion is contested by Lucey and Tully [10], who say that this relationship strengthens and weakens over time but is prevalent in the long run. Similarly, Sari et al. [20] notice a strong relationship between gold and silver. Śmiech and Papież [25] show that causality among the prices of gold, silver, platinum and copper change in the period 2000-2011. Also Papież and Śmiech [14] examine causality in mean and variance between commodity prices (including metal prices) and financial market prices.

The second area of analysis is connected with examining the volatility of returns of the precious metals. Hammoudeh and Yuan [5] show that gold and silver have similar volatility persistence globally, but there is no leverage effect in gold and silver prices. Sari et al. [21] examine the co-movements and information transmission among the spot prices of four precious metals (gold, silver, platinum, and palladium), the oil price, and the US dollar/euro exchange rate. Hammoudeh et al. [6] examine the conditional volatility and correlation dependence for four major precious metals, and they find that almost all of them are weakly responsive to news spilled over from other metals in the short run. Morales and Andreosso-O'Callaghan [11] find that in terms of volatility spillovers, an asymmetric effect is observed; gold tends to dominate the markets and the evidence favouring the case of other precious metals influencing the gold market is weak. Cochran et al. [3] show that events taking place during the post-September 2008 period increased the volatility in gold, platinum, and silver returns. Sensoy [22] claims that the turbulent year of 2008 had no significant effect on the volatility levels of gold and silver, although caused an upward shift in the volatility levels of palladium and platinum. Using the consistent dynamic conditional correlations, he shows that precious metals became strongly correlated with each other during the last decade, which reduces the diversification benefits across them and indicates a convergence to a single asset class.

The objective of this study to identify the states of the precious metals market (gold, silver, platinum and palladium) and to present their temporal evolution in the period from September 22, 1997 to February 13, 2014. Since our sample period covers the recent global financial crisis, we want to examine whether the market states are affected by the financial crisis. The process of identifying the states of the precious metals market and analysing their temporal evolution is based on the conditional dependence structure using a copula-DCC-GARCH methodology.

This allows us to address several questions, which might be of interest to both investors and researchers:

- Is the dependence among prices in the precious metals markets stable or does it undergo changes?

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- Are the changes in relations among precious metals prices evolutionary or drastic?
- What are the causes of drastic changes in relations among these prices?

The paper contributes to the existing literature in the following aspects.

Firstly, most analyses of the precious metals market conducted so far are based on standard multivariate GARCH (MGARCH) models (see e.g. Hammoudeh et al. [6], Morales and Andreosso-O'Callaghan [11], Sensoy [22], Silvennoinen and Thorp [24]), which assume that standardized innovations follow a multivariate elliptical distribution. In case of the multivariate normal distribution all marginal distributions must be normal, and multivariate Student's t distribution imposes, also often unrealistically, the same degrees of freedom for all marginal distributions.

However, a copula-based multivariate GARCH model used in this study allows for modelling the conditional dependence structure when standardized innovations are non-elliptically distributed. Thus, it makes it possible to model the volatility of particular metals using univariate GARCH models with different standarized residual distribution. Generally, copulas allow the researcher to specify the models for the marginal distributions separately from the dependence structure that links these distributions to form a joint distribution. They offer a greater flexibility in modelling and estimating margins than while using parametric multivariate distributions (see e.g. Nelsen [13], Joe [7]).

Secondly, at present a copula-GARCH methodology is widely used in the analysis of financial time series (see e.g. Aloui et al. [1], Lee and Long [8], Li and Yang [9], Patton [15], Philippas and Siriopoulos [17], Serban et al. [23], Wu et al. [26], Zolotko and Okhrin [27] and for a review Patton [16]). However, in most studies on the precious metals market, copula methodologies are used to analyse the dependencies among single metal markets and other markets (see e.g. Reboredo [18, 19]). This study is based on conditional correlations using a copula-GARCH methodology to investigate the dynamics of conditional dependence structure among precious metals. It also attempts to identify the states of the market on the basis of these conditional correlations and to follow their temporal evolution. To the best of our knowledge, such approach has not been applied to investigate dependencies in the precious metals markets so far.

The paper is organised as follows. Section 2 describes the data and econometric methodologies employed. Empirical results are discussed in Section 3, and the conclusions are presented in the last section.

2 Methodology

The dynamic relationship among precious metals is analysed with the use of a copula-DCC-GARCH model for daily log-returns. In this approach, multivariate joint distributions of the return vector $r_t = (r_{1,t},...,r_{k,t})'$, $t = 1,...,T$, conditional on the information set available at time $t-1$ (Ω_{t-1}) is modelled using conditional copulas introduced by Patton [15]. This model takes the following form:

$$r_{1,t} \mid \Omega_{t-1} \sim F_{1,t}(\cdot \mid \Omega_{t-1}),...,r_{k,t} \mid \Omega_{t-1} \sim F_{k,t}(\cdot \mid \Omega_{t-1}), \quad (1)$$

$$r_t \mid \Omega_{t-1} \sim F_t(\cdot \mid \Omega_{t-1}), \quad (2)$$

$$F_t(r_t \mid \Omega_{t-1}) = C_t\big(F_{1,t}(r_{1,t} \mid \Omega_{t-1}),...,F_{k,t}(r_{k,t} \mid \Omega_{t-1}) \mid \Omega_{t-1}\big), \quad (3)$$

where C_t denotes the copula, while F_t and $F_{i,t}$ respectively the joint cumulative distribution function and the cumulative distribution function of the marginal distributions at time *t*. Using a copula allows for separate modelling of marginal distributions and the dependence structure of vector r_t. In the empirical study elliptical copulas are used to describe the dynamics of the dependence structure, while conditional marginal distributions are modelled with the use of ARMA-GARCH models. Conditional correlation matrices R_t modelled with the use of DCC(1, 1) model (Engle [4]) are assumed to be the parameters of conditional copulas.

The states of the precious metals market are identified on the basis of conditional correlation matrices. It is assumed that a given market state corresponds to a typical pattern of the conditional dependence structure described by a conditional correlation matrix R_t. Transition points between market states, corresponding to drastic changes in the conditional dependence structure, are identified using Ward's method of cluster analysis and a similarity measure suggested by Münnix et al. [12], which allows us to quantify the difference of the correlation structure for two points in time.

3 Data and empirical results

The data used in this study consist of the daily (five working days per week) spot prices of gold (Gold), silver (Silv), platinum (Plat) and palladium (Pall) from September 22, 1997 to February 13, 2014. The source of data is Bloomberg, and all prices of precious metals are measured in US dollars per troy ounce. As usual, price return series are computed on a continuous compounding basis as $r_{i,t} = 100 \times (\log(P_{i,t} / P_{i,t-1}))$, where $P_{i,t}$ and $P_{i,t-1}$ are current and one-period lagged spot prices of precious metals. After eliminating the mismatching transaction days, we finally obtain 4185 log-returns for each series.

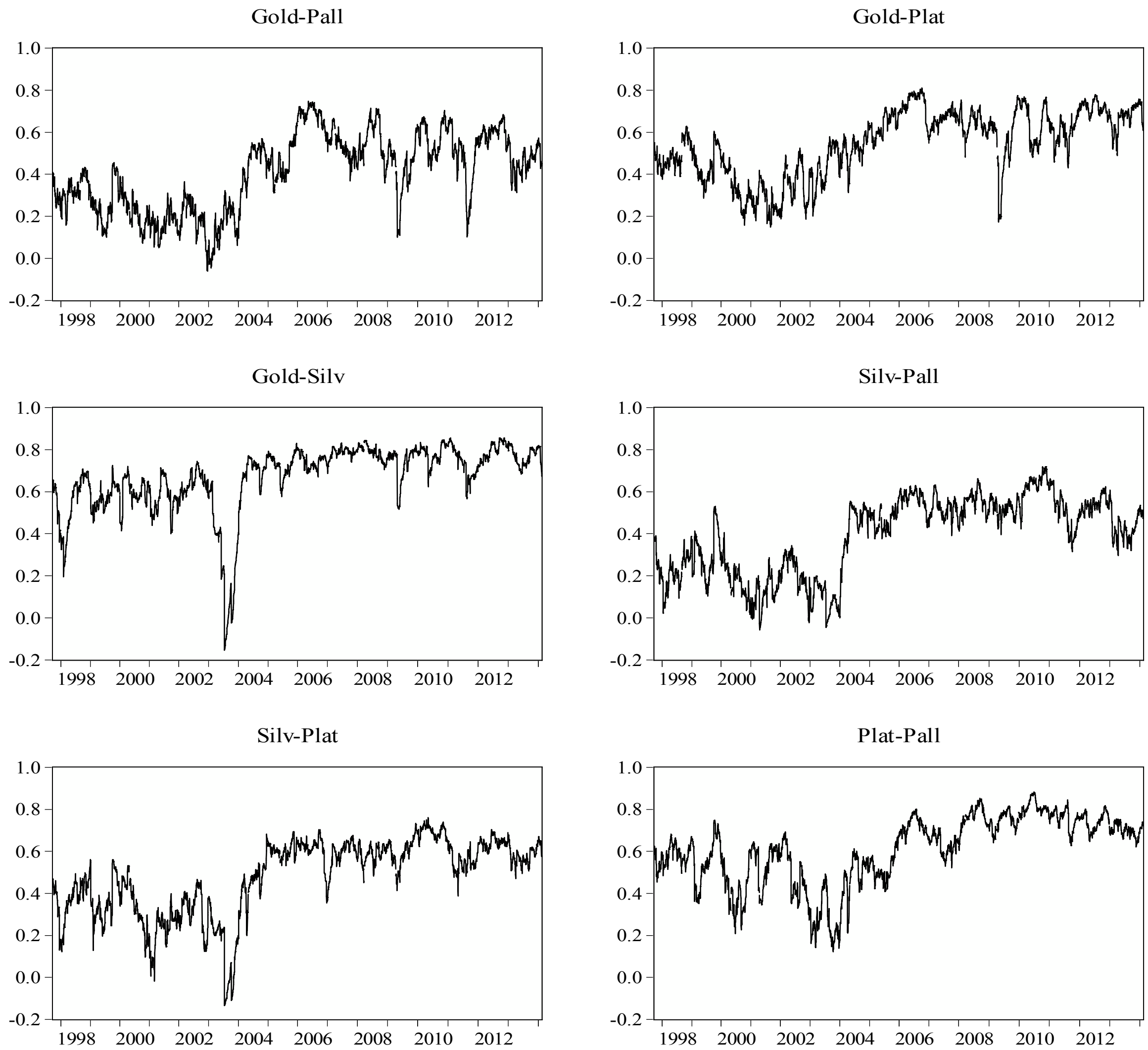

Figure 1 Dynamic correlations of precious metal returns

In the empirical study different variants of the AR-GARCH specification are considered for individual returns. Eventually, on the basis of information criteria, Student's t AR(1)-GARCH(1,1) model has been assumed for gold, normal AR(1)-GARCH(1,1) model has been assumed for silver, skewed Student's t ARMA(1,1)-GARCH(1,1) model has been assumed for platinum, and Student's t AR(1)-GARCH(1,1) has been assumed for palladium. On the other hand, Gauss and Student's t copulas have been considered in the analysis of the dynamics of dependencies among the rates of return, and, again on the basis of information criteria[4], Student's t has been chosen.

Conditional correlation matrices obtained with the use of the estimated model are applied to analyse the precious metals market. Dynamic correlations in this marked are presented in Fig. 1. Figure 2 shows the temporal evolution of the market states in the period from September 22, 1997 to February 13, 2014, obtained as a result

[4] The parameters of the model are assessed using R package ”rmgarch” (version 1.2-6), developed by Alexios Ghalanos. The results can be obtained from the author on request.

of clustering conditional correlation matrices with Ward's method of cluster analysis. The left panel illustrates the division into two clusters (Rousseeuw's Silhouette internal cluster quality index equals 0, 5512), while the right panel illustrates the division into three clusters (Rousseeuw's Silhouette internal cluster quality index equals 0,3169). Structural changes in the precious metals markets in the analysed period are identified with the use of a similarity measure of correlation matrices (Münnix et al. [12]) and presented in Fig. 3 (the left panel illustrates the similarity of matrices distant from each other by a multiple of one quarter, the right panel – by a multiple of one year). Light shading denotes similar conditional correlation matrices and dark shading denotes dissimilar ones. If we assume that a point on the diagonal designates "now", then the similarity to previous times from this point can be found on the vertical line below this point, or the horizontal line to the left of this point.

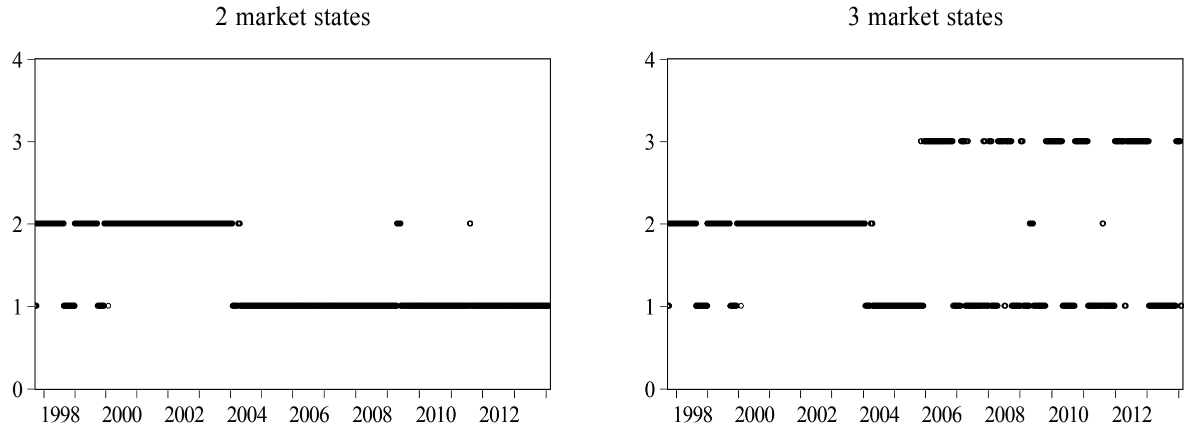

Figure 2 Temporal evolution of the market state

After analysing the dynamics of conditional correlation matrices (Fig. 1), it can be said that bilateral correlation among precious metals markets increased considerably in 2004 and has remained at this level since then. A similar conclusion can be drawn from the analysis of similarity maps showing conditional correlation matrices (Fig. 3). Particular areas in the matrix show how similar dependence measures in two periods are: the first one is on the horizontal axis and the second one on the vertical axis. Figure 3 presents the results for two options: the first option covers correlations calculated for sub-periods with 70 observations, while the second – for sub-periods with 250 observations. A low value of a similarity measure of correlation matrices (considerably darker shading) for 2004 together with its rise and maintaining this high level (lighter shading) indicate structural changes in precious metals markets in 2004. It confirms Sensoy's [22] hypothesis that precious metals will be a single asset class in the near future. This change in the precious metals markets in 2004 is also evident in clustering results (Fig. 2). Divisions into two and three clusters indicate a stable change of the market state in April 2004 (4/29/2004). For the last decade this market has not returned (with few exceptions) to the state from before 4/29/2004. The more detailed analysis of this decade (the division into 3 clusters) reveals two basis states with numerous transition points between them. On the basis of the results obtained, it can be concluded that the global financial crisis from 2008 has not considerably affected the precious metals market.

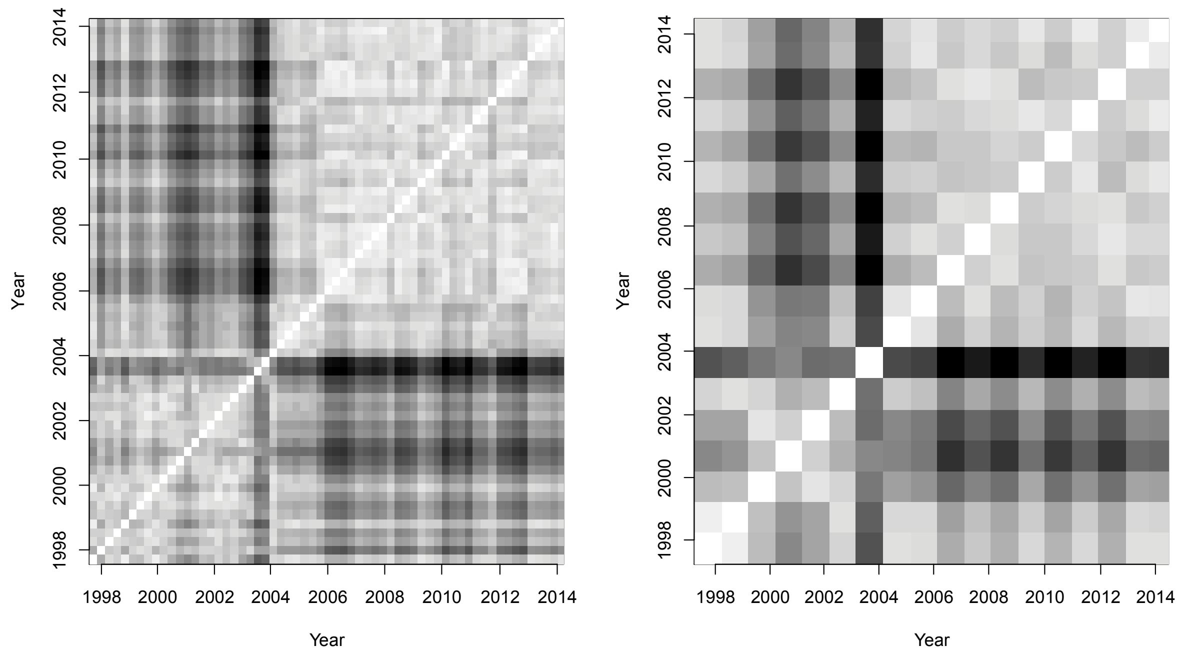

Figure 3 Correlation/similarity matrix for the precious metals market

4 Conclusion

The objective of this study is to analyse the conditional dependence structure among precious metals using the copula-DCC-GARCH methodology and to follow their temporal evolution. The results obtained in the study reveal that the dependence structure is not stable over time. Internal clustering criteria applied to Ward's method prove that two (which seems to be better choice) or three typical patterns of the conditional dependence are plausible. If two market states are assumed, the transition point takes place in April 2004. The state of the precious metals market before and after this moment is stable, with rare and transitory changes. Conditional correlations in the first period are lower than in the second period. If three market states are assumed, the one till April 2004 is stable, but later we observe two patterns which change frequently. The similarity between this two last state are however quite high. Summing up, the results obtained indicate that the dependence structure of precious metals undergoes only one drastic structural change in April 2004. It confirms Sensoy's [22] hypothesis that precious metals will be a single asset class in the near future. A unique opportunity to test this thesis was the global financial crisis, which, however, did not affect the correlation structure of precious metals returns. The results obtained might be of great importance to investors, as they demonstrate that drastic changes of the correlation structure of the precious metals market is currently highly unlikely.

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